§ 13.3–Double Integrals in Polar Coordinates

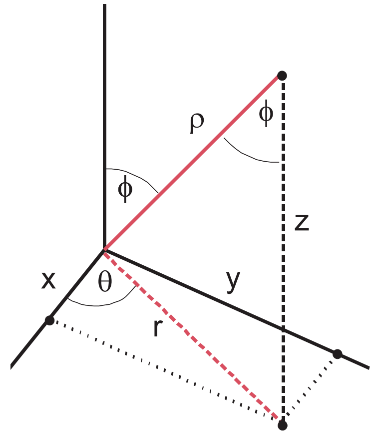
This lesson covers the material in Section 13.3

Read Lesson 20 in the Study Guide and Section 13.3 in the text.

Continue working on online homework.

Try: 7, 9, 13, 17, 19, 21, 23, 27, 31, 37, 43

Polar Coordinates



$x = r\cos\theta \quad y = r\sin\theta$

$x^2 + y^2 = r^2$

$\tan\theta = \dfrac{y}{x}$

Polar Rectangles

A *polar* rectangle R is a region in the $xy-$plane given by

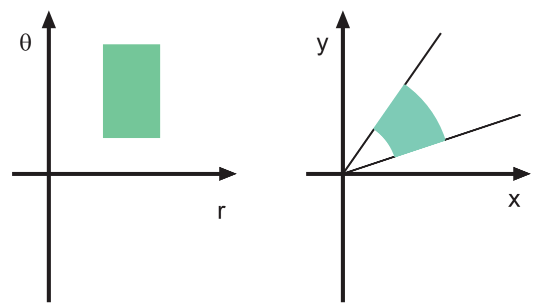
$$R = \{(r,\theta) \mid a \le r \le b,\ \alpha \le \theta \le \beta\}$$

where (r,θ) are polar coordinates.
If $f(x,y)$ is continuous on a polar rectangle R, and if $\beta - \alpha \le 2\pi$, then the double integral of f over R can be evaluated as

$$\iint_R f(x,y)\,dA = \int_\alpha^\beta \int_a^b f(r\cos\theta, r\sin\theta)\,r\,dr\,d\theta$$

Why?

Integration Factor

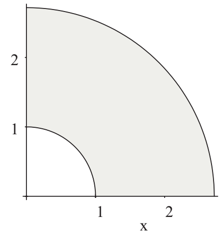


Compare areas: $\Delta\theta \cdot \Delta r$ vs.

$$\frac{\pi(r_2^2 - r_1^2)(\Delta\theta)}{2\pi} = \overline{r} \cdot \Delta\theta \cdot \Delta r$$

Examples

- Evaluate the double integral $\iint_R \frac{\ln(\sqrt{x^2+y^2})}{x^2+y^2}\, dA$ where R is the region in the first quadrant between the unit circle $x^2+y^2=1$ and the circle of radius e, $x^2+y^2=e^2$, that is shown below.

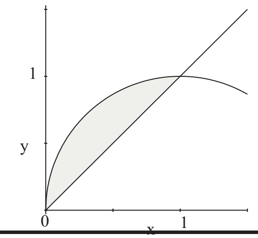


Examples, II

- Evaluate the double integral

$$\iint_R 6y\, dA$$

where R is the region in the first quadrant bounded above by the circle $(x-1)^2+y^2=1$ and below by the line $y=x$.



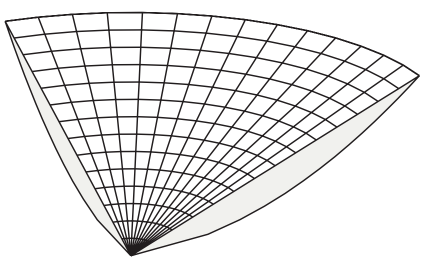
Examples, III

- Use polar coordinates to find the volume of the solid between the paraboloid

$$z = 3x^2 + 3y^2$$

and the circular cone

$$z = 6\sqrt{x^2+y^2}$$



Examples IV

- Evaluate the double integral

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \frac{1}{\sqrt{(x^2+y^2)\tan^{-1}(y/x)}}\, dy\, dx$$